Efficiency of Electron-Positron Pair Production by Neutrino Flux from Accretion Disk of a Kerr Black Hole

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Abstract

Dominant processes of neutrino production and neutrino-induced e^+e^--pair production are examined in the model of a disk hyper-accreting onto a Kerr black hole. The efficiency of plasma production by a neutrino flux from the disk, obtained for the both cases of presence and absence of a magnetic field, is found to be no more than several tenths of percent and, therefore, not enough for the origin of cosmological gamma-ray bursts.

The origin of the gamma-ray burst is among the most important problems to be solved. Various observations are in good agreement with a phenomenological model implying that gamma-ray bursts are produced by an ultrarelativistic e^+e^--plasma jet (fireball) [1]. Observations indicate that gamma-ray bursts vary rapidly and some of them arrive from cosmological distances. This makes to suggest that the fireball is produced in a compact region and has a huge energy of $\mathcal{E} \gtrsim 10^{51}$ erg [2]. One of the natural sources of such a fireball could be neutrinos being able to carry away up to ten percent of the gravitational energy released in a collapse in compact systems. Taking into account the smallness of weak-interaction cross sections one can expect that only a small fraction of the energy released is transferred to the e^+e^--plasma, what in fact is enough to produce the fireball with the energy pointed. However, the plasma produced can go out and remain the ultrarelativistic one (what is necessary for its further transformation into the observed gamma-ray burst) in a region with a sufficiently low baryon density [3]

The conditions discussed can be realized in systems involving an accretion disk around a Kerr black hole, e.g., failed supernova [4], collapsar with hyper-accretion [5], and hypernova [6]. Due to the high accretion velocities and viscosity, the density and temperature of the inner part of the disk can be as high as $\rho \sim 10^{10} - 10^{11}$ g/cm^3 and $T \sim 5 - 10$

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MeV, so that neutrino luminosity reaches the value $L_\nu \sim 10^{53}$ erg/s. At the same time, a region of a low baryon density can be formed in the vicinity of the rotation axis [4, 5]. Thus, a large neutrino flux from the disk generates the plasma which can go out with the energy sufficient for producing a gamma-ray burst.

It is important to note that strong magnetic fields can exist in the accretion disk. The field strength in the viscous disk of the densities we are interested in can reach values [7]:

$$B \lesssim 10^{15}G \left(\frac{\alpha}{0.1}\right)^{1/2} \left(\frac{c_s}{10^9 cm/s}\right) \left(\frac{\rho}{10^{11}g/cm^3}\right)^{1/2}, \qquad (1)$$

where α is the dimensionless viscosity parameter and c_s is the speed of the sound. The magnetic field can have a rather complicated structure, however, for the processes considered only the field strength is important.

The main process to create plasma by the neutrino flux in a rarefied medium is considered as:

$$\nu_i + \tilde{\nu}_i \Longrightarrow e^+ + e^- \quad (i = e, \mu, \tau), \qquad (2)$$

It has been studied to apply to various astrophysical processes. In early works, its influence on the explosion dynamics of a type-II supernova was examined. For this purpose its luminosity in e^+e^- -pairs in the simplest models of neutrino blackbody emission to the vacuum [8] and later on taking into account the fact that the neutrino flux goes through a partially transparent medium of the shell [9]. As the evidences in favor of the cosmological origin of the gamma-ray burst were accumulated, the process (2) has been considered as a possible energy source of the fireball [10]. The detailed numerical calculations of the fireball production were performed only in recent papers [4, 5, 11], however the magnetic field influence on plasma production was not taking into account in these papers. Let us note that such an influence can be substantial one in the strong field. Indeed, in this case new reactions of the e^+e^- -pair production:

$$\nu_i \Longrightarrow \nu_i + e^+ + e^-, \qquad (3)$$
$$\tilde{\nu}_i \Longrightarrow \tilde{\nu}_i + e^+ + e^- \qquad (4)$$

are not only opened kinematically but can dominate as well. The importance of these processes as a possible energy source of a cosmological gamma-ray burst was pointed out first in the paper [12].

In the present paper we use the model of a disk hyper-accreting onto a Kerr black hole to estimate the efficiency of e^+e^- -plasma production in processes involving neutrinos. The efficiency is defined as the ratio of the e^+e^- -pair luminosity $L_{e^+e^-}$ (energy emitted per unit time) to the neutrino luminosity $(L_\nu + L_{\tilde{\nu}})$ from the disk:

$$\epsilon = L_{e^+e^-}/L_{tot}, \quad L_{tot} = L_\nu + L_{\tilde{\nu}}. \qquad (5)$$

This paper focuses on deriving analytical expressions for the efficiency of plasma production in the dominant neutrino processes in a simplified model of accretion disk taking into account a strong magnetic field. It's natural to expect that the efficiency will be

estimated only. However such an approach would allow to show how the luminosity depends on the system parameters. We also neglect general relativity effects on plasma production. It's known, that the gravitational field can influence in two different ways: the neutrino redshift reduces the e^+e^- -pair luminosity, where as the bending of neutrino trajectories increases it due to an increase in the collision frequency. An analysis of these effects indicates that the bending effect dominates only at sufficiently large radii of the last Keplerian orbit ($R_0 \gtrsim 3r_g$, where r_g is the gravitational radius of a black hole) [13]. However, the efficiency even in this case increases no more than by a factor of 2.

As it was mentioned above, to ensure the required neutrino luminosity, the inner part of the disk should have high densities and temperatures. Such parameters can be attained in hyper-accretion onto a Kerr black hole [5]. Leaving aside the question of stability of a system with such accretion rates, we use the results of [5] to determine the disk parameters. The gradients of density and temperature pointed in [5] allow us to consider the neutrino-emitting part of the disk is uniform. At an accretion rate of $\dot{M} \sim 0.1 M_\odot/$s, the typical densities and temperatures are $\rho \sim 10^{11}$ g/cm^3 and $T \sim 5$ MeV, respectively. For these parameters, neutrinos are predominantly emitted by Urca processes

$$p + e^- \Longrightarrow n + \nu_e, \tag{6}$$
$$n + e^+ \Longrightarrow p + \tilde{\nu}_e. \tag{7}$$

An analysis indicates that a magnetic field of $B \sim 10^{15}$ G has negligible effect on the cross sections for these reactions. The neutrino mean free path in such a medium is estimated as

$$l_\nu \sim 10 \; km \left(\frac{10^{11} g/cm^3}{\rho} \right) \left(\frac{5 MeV}{T} \right)^2. \tag{8}$$

Thus, the disk part under consideration can be treated as transparent to neutrinos. Note that the typical times of establishing the β-equilibrium in Urca processes (6) and (7) are $\tau_\beta \sim 10^{-2}$ s for the medium parameters used. The characteristic dynamical accretion time can roughly be estimated as the time it takes for a nucleon flux to pass through the neutrino-emitting disk part and is also about $\tau_d \sim 10^{-2}$ s. Thus, the accreting matter does not arrive at the β-equilibrium and, therefore, the parameter $Y = N_p/(N_p + N_n)$ is indeterminate and can vary in the interval

$$Y_\beta < Y < 0.5, \tag{9}$$

where N_p and N_n are the proton and neutron number densities in the disk and Y_β is the Y parameter at β-equilibrium ($Y_\beta \sim 0.1$ for the densities and temperatures under consideration).

Because the neutrino mean free path exceeds the characteristic transverse dimension of the disk, neutrinos are free streaming throughout the disk. In this case, the neutrino luminosity is calculated in the standard way by using the Lagrangian for the interaction of charged electron-neutrino and nucleon currents in the low-energy approximation [14] and can be represented as

$$L_{\nu,\tilde{\nu}} = \int \omega F_{\nu,\tilde{\nu}} d^3 n, \tag{10}$$

$$F_{\nu,\tilde{\nu}} = \frac{G_F^2 \cos^2\theta_c(1+3g_a^2)}{\pi} \frac{\omega^2 N_{p,n}}{\exp[\omega/T \mp a]+1}. \tag{11}$$

Here ω is the neutrino energy, d^3n is the neutrino phase-space element, T is the temperature of the medium, $a = (\mu - m_n + m_p)/T$, where μ is the electron chemical potential, m_n and m_p are the neutron and proton masses, respectively, N_n and N_p are the neutron and proton number densities, respectively, g_a is the axial constant of the charged nucleon current ($g_a \simeq 1.26$ in the low-energy limit), G_F is the Fermi constant and θ_c is the Cabibbo angle. Integral (10) can easily be calculated for the simplified model of a uniform disk. As a result, the (anti)neutrino luminosity in the Urca processes from the disk is written as

$$L_{\nu,\tilde{\nu}} = \frac{(G_F \cos\theta_c)^2}{2\pi^3}(1+3g_a^2)N_{p,n}T^6 V I_5(\pm a), \quad I_s(a) = \int\limits_0^\infty \frac{y^s dy}{\exp(y-a)+1}, \tag{12}$$

where V is the emitting disk volume. The ratio of the neutrino luminosity in the Y interval (9) is $L_{\tilde{\nu}}/L_\nu \ll 1$. As is seen in the figure, even in the most favorable case of β-equilibrium, this ratio is about one tenth and decreases very rapidly as Y increases. Thus, we set $L_{tot} \simeq L_\nu$ in all cases unless this will lead to confusion.

We calculate the e^+e^- -pair luminosity for the case where the magnetic field is sufficiently strong but the parameter eB is much less than the neutrino mean energy squared:

$$m_e^2 \ll eB \ll \omega^2, \tag{13}$$

which is satisfied well in the case under consideration. Here, m_e is the electron mass and $e > 0$ is the elementary charge. As was argued above, it is most important to estimate the plasma production efficiency within a small solid angle around the system rotation axis. Because the medium in this cone is rarified, its effect on the processes can be neglected. The magnetic field can have a complex structure in this region, but we treat its field lines as directed along the rotation axis.

The electron-positron pair emissivity per unit volume in reaction (2) is determined by

$$Q_{\nu\tilde{\nu}\to e^+e^-} = \int j\sigma q_0 dN_\nu dN_{\tilde{\nu}}, \quad dN_{\nu,\tilde{\nu}} = \frac{\omega^2 F_{\nu,\tilde{\nu}}}{8\pi^3 R^2} dV d\omega. \tag{14}$$

where σ is the cross section for the process, $j = q^2/(2\omega_1\omega_2)$ is the relative velocity in the rest frame of one of the colliding particles, $dN_{\nu,\tilde{\nu}}$ is the (anti)neutrino number density at a distance R from the element dV of the isotropically emitting disk, and $q = q_1 + q_2$ is the 4-momentum transfer in the reaction. An analysis shows that the magnetic field only slightly affects the cross section in the approximation (13):

$$\sigma = \sigma_0\left(1 + O\left(\frac{eB}{\omega^2}\right)\right), \quad \sigma_0 = \frac{G_F^2}{3\pi}\left(c_v^2 + c_a^2\right)q^2, \tag{15}$$

where σ_0 is the cross section for the process in vacuum, $c_v = 1/2 + 2\sin^2\theta_W \simeq 0.96$ and $c_a = 1/2$ are, respectively, the vector and axial constants of the charged neutrino-electron current and θ_W is the Weinberg angle ($\sin^2\theta_W \simeq 0.23$). Therefore, the luminosity in

the reaction under study can be estimated in the vacuum approximation. By integrating Eq. (14) over the volume of a cone with a solid angle of $\Delta\Omega << 4\pi$ along the black hole rotation axis, we obtain the formula for the e^+e^- -pair luminosity. It is reasonable to relate this expression to the neutrino and antineutrino luminosity from the inner part of the disk:

$$L_{\nu\tilde{\nu}\to e^+e^-} = \frac{G_F^2(c_v^2 + c_a^2)}{128\pi}L_\nu L_{\tilde{\nu}}\frac{T}{R_0}\left(\frac{\Delta\Omega}{4\pi}\right)\left[\frac{I_6(a)}{I_5(a)} + \frac{I_6(-a)}{I_5(-a)}\right], \tag{16}$$

where R_0 is the radius of the last Keplerian orbit.

The luminosity for processes (3) and (4) can be calculated by using the expression obtained in [15] for the rate of energy transfer to e^+e^- -plasma per one neutrino. In approximation (13), this expression can be represented with logarithmic accuracy as

$$\dot{E} = \frac{7G_F^2(c_v^2 + c_a^2)}{432\pi^3}(eB\omega\sin\theta)^2\ln\left[eB\omega\sin\theta/m_e^3\right], \tag{17}$$

where θ is the angle between the initial neutrino momentum and the magnetic field. By integrating this formula over neutrino distribution dN_ν (14) and cone volume, we obtain the total luminosity from the disk for the e^+e^- -plasma produced in process (3):

$$L_{\nu\to\nu e^+e^-} = \frac{7G_F^2(c_v^2 + c_a^2)}{1728\pi^2}L_\nu(eB)^2TR_0\left(\frac{\Delta\Omega}{4\pi}\right)\frac{I_6(a)}{I_5(a)}\ln\left[eBT/m_e^3\right]. \tag{18}$$

The antineutrino luminosity in process (4) is determined by the same formula with replacement

$$L_\nu \to L_{\tilde{\nu}}, \quad a \to -a. \tag{19}$$

Because the ratio $L_{\tilde{\nu}}/L_\nu$ is small (see figure), antineutrino reaction (4) makes small contribution to the total plasma luminosity.

Can new reactions (3) and (4) be competitive with basic process (2) of plasma production? The luminosity ratio for these processes can be written as

$$\frac{L_{\nu\to\nu e^+e^-}}{L_{\nu\tilde{\nu}\to e^+e^-}} = \eta\left(\frac{eB}{T^2}\right)^2\left(\frac{l_{\tilde{\nu}}}{R_0}\right), \tag{20}$$

where η is a dimensionless constant of the order of unity. Therefore, both processes can make comparable contributions to the e^+e^- -pair luminosity for the disk parameters used. However, the new processes become efficient only if magnetic fields (1) attain their maximum strengths in the disk–black hole system.

The efficiency of plasma production in process (2) is numerically estimated as

$$\epsilon_{\nu\tilde{\nu}\to e^+e^-} \simeq 10^{-2}\left(\frac{L_{\tilde{\nu}}}{L_\nu}\right)\left(\frac{\Delta\Omega}{4\pi}\right)\left(\frac{L_{tot}}{10^{53}erg}\right)\left(\frac{T}{5MeV}\right)\left(\frac{30km}{R_0}\right). \tag{21}$$

This expression depends strongly on the chemical composition of the medium through the ratio $L_{\tilde{\nu}}/L_\nu$ (see figure) and decreases rapidly from its maximum value as the medium deviates from the β-equilibrium. Thus, the efficiency of plasma production in the absence

of magnetic field does not exceed several tenths of percent and becomes negligible at essential deviation of the nucleon medium from β-equilibrium.

Similar estimation for process (3) yields

$$\epsilon_{\nu \to \nu e^+ e^-} \simeq 2 \times 10^{-3} \left(\frac{\Delta\Omega}{4\pi} \right) \left(\frac{B}{4 \times 10^{15} G} \right)^2 \left(\frac{T}{5 MeV} \right) \left(\frac{R_0}{30 km} \right). \qquad (22)$$

It is easy to see that the efficiency in this process is independent of the disk chemical composition. This implies that the plasma production by a single neutrino in a strong magnetic field may prevail over the annihilation if there is a deviation from the β-equilibrium. However, even in this case the efficiency of plasma production does not exceed several tenths of percent and decreases quadratically as the magnetic field decreases. Thus, the neutrino mechanism of plasma production in collapsing systems with hyper-accretion is likely to be inefficient. Ruffert and Janka [11] arrived at the similar conclusion for the model of close binary system merging into a black hole.

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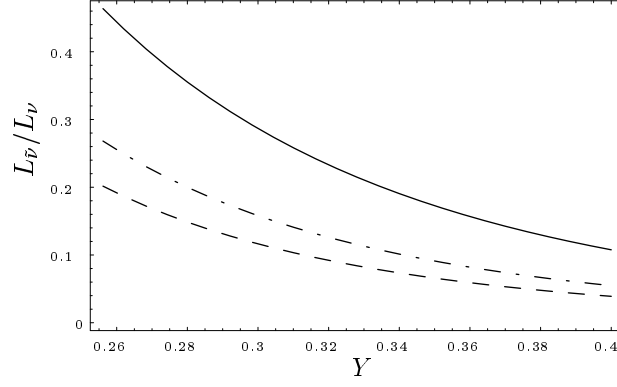[15] A. V. Kuznetsov and N. V. Mikheev, Phys. At. Nucl. **60**, 1865 (1997).

Figure 1: Neutrino luminosities ratio $L_{\tilde{\nu}}/L_{\nu}$ as function of parameter Y with fixed ρ and T. The solid line corresponds to $\rho = 10^{11}\ g/cm^3$, $T = 7\ MeV$, $Y_\beta = 0.30$; the dash-dotted line corresponds to $\rho = 5 \times 10^{10}\ g/cm^3$, $T = 5\ MeV$, $Y_\beta = 0.26$; dotted lines corresponds to $\rho = 10^{11}\ g/cm^3, T = 6\ MeV$, $Y_\beta = 0.24$.